OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4722

Core Mathematics 2

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

Registered Charity Number: 1066969
[Turn over

1 The 20th term of an arithmetic progression is 10 and the 50th term is 70.

(i) Find the first term and the common difference. [4]

(ii) Show that the sum of the first 29 terms is zero. [2]

2 Triangle ABC has $AB = 10\,\text{cm}$, $BC = 7\,\text{cm}$ and angle $B = 80°$. Calculate

(i) the area of the triangle, [2]

(ii) the length of CA, [2]

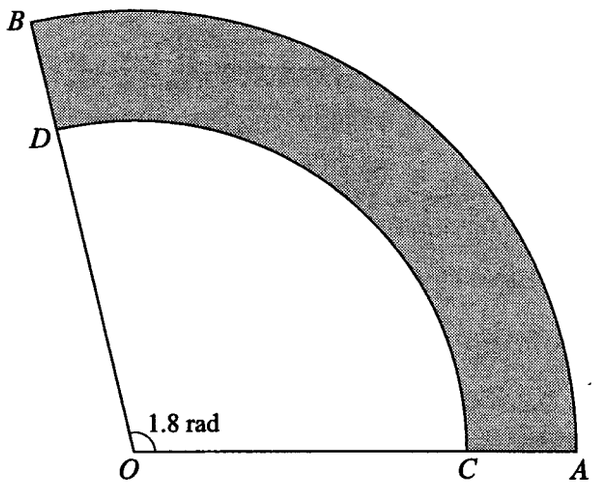
(iii) the size of angle C. [2]

3 **(i)** Find the first three terms of the expansion, in ascending powers of x, of $(1 - 2x)^{12}$. [3]

(ii) Hence find the coefficient of x^2 in the expansion of

$$(1 + 3x)(1 - 2x)^{12}.$$ [3]

4



The diagram shows a sector OAB of a circle with centre O. The angle AOB is 1.8 radians. The points C and D lie on OA and OB respectively. It is given that $OA = OB = 20\,\text{cm}$ and $OC = OD = 15\,\text{cm}$. The shaded region is bounded by the arcs AB and CD and by the lines CA and DB.

(i) Find the perimeter of the shaded region. [3]

(ii) Find the area of the shaded region. [3]

5 In a geometric progression, the first term is 5 and the second term is 4.8.

(i) Show that the sum to infinity is 125. [2]

(ii) The sum of the first n terms is greater than 124. Show that

$$0.96^n < 0.008,$$

and use logarithms to calculate the smallest possible value of n. [6]

6 **(a)** Find $\int \left(x^{\frac{1}{2}} + 4\right) \mathrm{d}x$. [4]

(b) **(i)** Find the value, in terms of a, of $\int_1^a 4x^{-2} \, \mathrm{d}x$, where a is a constant greater than 1. [3]

(ii) Deduce the value of $\int_1^\infty 4x^{-2} \, \mathrm{d}x$. [1]

7 **(i)** Express each of the following in terms of $\log_{10} x$ and $\log_{10} y$.

(a) $\log_{10}\left(\frac{x}{y}\right)$ [1]

(b) $\log_{10}(10x^2y)$ [3]

(ii) Given that

$$2\log_{10}\left(\frac{x}{y}\right) = 1 + \log_{10}(10x^2y),$$

find the value of y correct to 3 decimal places. [4]

8 The cubic polynomial $2x^3 + kx^2 - x + 6$ is denoted by $\mathrm{f}(x)$. It is given that $(x + 1)$ is a factor of $\mathrm{f}(x)$.

(i) Show that $k = -5$, and factorise $\mathrm{f}(x)$ completely. [6]

(ii) Find $\int_{-1}^2 \mathrm{f}(x) \, \mathrm{d}x$. [4]

(iii) Explain with the aid of a sketch why the answer to part **(ii)** does not give the area of the region between the curve $y = \mathrm{f}(x)$ and the x-axis for $-1 \leqslant x \leqslant 2$. [2]

[Question 9 is printed overleaf.]

9 **(i)** Sketch, on a single diagram showing values of x from $-180°$ to $+180°$, the graphs of $y = \tan x$ and $y = 4\cos x$. [3]

The equation

$$\tan x = 4\cos x$$

has two roots in the interval $-180° \leqslant x \leqslant 180°$. These are denoted by α and β, where $\alpha < \beta$.

(ii) Show α and β on your sketch, and express β in terms of α. [3]

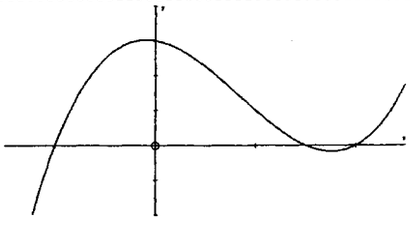
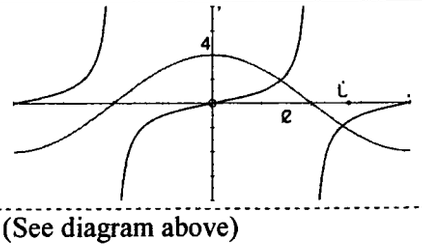
(iii) Show that the equation $\tan x = 4\cos x$ may be written as

$$4\sin^2 x + \sin x - 4 = 0.$$

Hence find the value of $\beta - \alpha$, correct to the nearest degree. [6]

1	(i)	$a+19d=10,\quad a+49d=70$	M1		Attempt to find d from simultaneous equations involving $a+(n-1)d$ or equiv method
		Hence $30d=60\Rightarrow d=2$	A1		Obtain $d=2$
		$a+(19\times 2)=10$ or $a+(49\times 2)=70$	M1		Attempt to find a from $a+(n-1)d$ or equiv
		Hence $a=-28$	A1	4	Obtain $a=-28$
	(ii)	$S=\frac{29}{2}(2\times -28+(29-1)\times 2)=0$	M1		For relevant use of $\frac{1}{2}n(2a+(n-1)d)$
			A1	2	For showing the given result correctly **AG**
				6	
2	(i)	$\Delta=\frac{1}{2}\times 10\times 7\times\sin 80=34.5\text{cm}^2$	M1		For use of $\frac{1}{2}ca\sin B$ or complete equiv.
			A1	2	For correct value 34.5
	(ii)	$b^2=10^2+7^2-2\times 10\times 7\times\cos 80$	M1		For attempted use of the correct cosine formula
		Hence length of CA is 11.2 cm	A1	2	For correct value 11.2
	(iii)	$\sin C=\frac{10\sin 80}{11.166...}=0.8819...$	M1		For use of the sine rule to find C, or equivalent
		Hence angle C is 61.9°	A1	2	For correct value 61.9
				6	
3	(i)	$(1-2x)^{12}=1-24x+264x^2$	B1		Obtain 1 and $-24x$ ...
			M1		Attempt x^2 term, including attempt at binomial coeff.
			A1	3	Obtain ...$264x^2$
	(ii)	$(1\times 264)+(3\times -24)=192$	M1		Attempt coefficient of x^2 from two pairs of terms
			A1√		Obtain correct unsimplified expression
			A1	3	Obtain 192
				6	
4	(i)	perimeter $=(15\times 1.8)+(20\times 1.8)+5+5$	M1		Use $r\theta$ at least once
		$=73\text{cm}$	A1		Obtain at least one of 27cm or 36cm
			A1	3	Obtain 73
	(ii)	$\text{area}=\left(\frac{1}{2}\times 20^2\times 1.8\right)-\left(\frac{1}{2}\times 15^2\times 1.8\right)$	M1		Attempt area of sector using $kr^2\theta$
			M1		Find difference between attempts at two sectors
		$=157.5\text{cm}^2$	A1	3	Obtain 157.5 / 158
				6	

5	(i)	$r=\frac{4.8}{5}=0.96\Rightarrow S_\infty=\frac{5}{0.04}=125$	B1* B1 dep * **2**		For correct value of r used For correct use of $\frac{a}{1-r}$ to show given answer **AG**
	(ii)	$S_n=\frac{5(1-0.96^n)}{1-0.96}$ Hence $1-0.96^n>0.992\Rightarrow 0.96^n<0.008$ $n\log 0.96<\log 0.008$ Hence $n>\frac{\log 0.008}{\log 0.96}\approx 118.3$ Least value of n is 119	B1 M1 A1 B1 M1 A1 **6** **8**		For correct, unsimplified, S_n For linking S_n to 124 (> or =) and multiplying through by 0.04, or equiv. For showing the given result correctly, with correct inequality throughout **AG** For correct log statement seen or implied (ignore sign) For dividing both sides by log 0.96 For correct (integer) value 119
6	(a)	$\frac{2}{3}x^{\frac{3}{2}}+4x+c$	M1 A1 B1 B1 **4**		For $kx^{\frac{3}{2}}$ For correct first term $\frac{2}{3}x^{\frac{3}{2}}$, or equiv For correct second term $4x$ For $+c$
	(b)(i)	$\int_1^a 4x^{-2}dx=\left[-4x^{-1}\right]_1^a$ $=4-\frac{4}{a}$	M1 M1 A1 **3**		Obtain integral of the form kx^{-1} Use limits $x=a$ and $x=1$ Obtain $=4-\frac{4}{a}$, or equivalent
	(ii)	4	B1√ **1** **8**		State 4, or legitimate conclusion from their (b)(i)
7	(i)(a)	$\log_{10}x-\log_{10}y$	B1 **1**		For the correct answer
	(b)	$1+2\log_{10}x+\log_{10}y$	M1 A1 A1 **3**		Sum of three log terms involving 10, x^2, y For correct term $2\log_{10}x$ For both correct terms 1 and $\log_{10}y$
	(ii)	$2\log_{10}x-2\log_{10}y=2+2\log_{10}x+\log_{10}y$ Hence $3\log_{10}y=-2$ So $y=10^{-\frac{2}{3}}\approx 0.215$	M1 A1 M1 A1 **4** **8**		For relevant use of results from (i) For a correct, unsimplified, equation in $\log_{10}y$ only For correct use of $a=\log_{10}c\Leftrightarrow c=10^a$ For the correct value 0.215

8	(i)	$-2+k+1+6=0 \Rightarrow k=-5$	M1		For attempting f(–1)
			A1		For equating f(-1) to 0 and deducing the correct value of k **AG**
		OR	M1		Match coefficients and attempt k
			A1		Show k = -5
		OR	B2		Following division, state remainder is 0, hence $(x + 1)$ is a factor, hence k = -5
		EITHER: $(x+1)(2x^2-7x+6)$	B1		For correct leading term $2x^2$
			M1		For attempt at complete division by f(x) by $(x + 1)$ or equiv.
			A1		For completely correct quadratic factor
		$=(x+1)(x-2)(2x-3)$	A1		For all three factors correct
		OR: f(2) = 16 – 20 – 2 + 6 = 0	M1		For further relevant use of the factor theorem
		Hence $(x-2)$ is a factor			
		Third factor is $(2x-3)$	A1		For correct identification of factor $(x-2)$
		Hence f(x) $=(x+1)(x-2)(2x-3)$	M1		For any method for the remaining factor
			A1	6	For all three factors correct
	(ii)	$\int_{-1}^{2} f(x)dx = \left[\frac{1}{2}x^4-\frac{5}{3}x^3-\frac{1}{2}x^2+6x\right]_{-1}^{2}$	B1√		For any two terms integrated correctly
			B1√		For all four terms integrated correctly
		$=\left(8-\frac{40}{3}-2+12\right)-\left(\frac{1}{2}+\frac{5}{3}-\frac{1}{2}-6\right)$	M1		For evaluation of F(2) – F(–1)
		$=9$	A1	4	For correct value 9
	(iii)		B1		For sketch of positive cubic, with three distinct, non-zero, roots
			B1	2	For correct explanation that some of the area is below the axis
				12	
9	(i)		B1		For correct sketch of one curve
			B1		For correct shape and location of second curve, on same diagram
			B1	3	For intercept 4 on y-axis
	(ii)	(See diagram above)	B1		For correct identification of intersections – in correct order
		$\beta = 180 - \alpha$	M1		For attempt to use symmetry of the graphs
			A1	3	For the correct (explicit) answer for β
	(iii)	$\sin x = 4\cos^2 x = 4(1-\sin^2 x)$	M1		For use of $\tan x = \frac{\sin x}{\cos x}$
			M1		For use of $\cos^2 x = 1-\sin^2 x$
		Hence $4\sin^2 x+\sin x-4=0$	A1		For showing the given equation correctly
		$\sin x = \frac{-1\pm\sqrt{65}}{8}$	B1		For correct solution of quadratic
		Hence $\beta-\alpha = 118.02...-61.97... \approx 56^\circ$	M1		Attempt value for x from their solutions
			A1	6	For the correct value 56
				12	